Estimation of fatigue life of selected construction materials under cyclic loading

M. Kurek, T. Łagoda
Opole University of Technology, Poland
ma.kurek@po.opole.pl, t.lagoda@po.opole.pl

S. Vantadori
University of Parma, Italy
sabrina.vantadori@unipr.it

ABSTRACT. In the literature, there are many criteria of multiaxial fatigue. They are based on various assumptions and parameters describing the process of fatigue. Among them, there is a special group of criteria based on the concept of critical plane. Some of them in their equations take into account the ratio of normal and shear stresses. Macha has formulated the criterion of maximum normal and shear stress in fracture plane which can be generalised for the scope of random loading of numerous criteria. In the present study authors estimated the fatigue life of several construction materials. For the purposes of the analysis, the authors proposed modified Carpinteri et al. method to find orientation of critical plane, which is used in multiaxial fatigue criterion defined in critical plane. This plane is turned through the angle of β in relation to the plane defined by maximum normal stresses. In this study authors analyzed the variability calculation of fatigue life, depending on the angle β. Simulation studies were conducted in which it was assumed that $\beta \in <0°, 45°>$. For each of the 46 angles, we calculated parameters B and K appearing in the formula defining equivalent stress. Then we calculated fatigue life according to the proposed model for each of the obtained angles β. Fatigue life analysis was carried out in order to verify which angle β gives the most similar results.

KEYWORDS. Fatigue life; Multiaxial criteria.

INTRODUCTION

The phenomenon of fatigue of materials and structures is a significant issue of the day. Fatigue occurs in various fields of industry, i.e. aerospace, machinery, mining or transport. The main objective of the bulk of research on predicting the fatigue strength is to identify a method of estimating the fatigue strength already on the stage of design and construction of components of machines and devices. The fatigue criterion in multiaxial loading is based on establishing such equivalent value that would enable comparison of multiaxial load with uniaxial loading. Literature of the subject provides a number of multiaxial fatigue criteria [6]. Such criteria are based on various assumptions and parameters of the fatigue process. A separate group of criteria among them are based on the critical plane concept [4]. Some of the criteria include the ratio of bending fatigue to torsional fatigue. The paper discusses estimation of fatigue strength depending on the changing orientation of the critical plane of proportional torsional bending for specific construction materials. The paper also compares the calculation and experimental results for fatigue strength of specific materials, using multiaxial fatigue criteria and various methods for determining critical plane orientation that take into account the ratio of fatigue limits.

FATIGUE STRENGTH ALGORITHM

To estimate calculation fatigue life used standard model, which consists of several stages. The first step includes measurement, generation or calculation of component of stress tensor, according to the following equations:

$$\sigma_{xx}(t) = \sigma_a \sin(\omega t) \tag{1}$$

$$\tau_{xy}(t) = \tau_a \sin(\omega t - \phi) \tag{2}$$

where:
σ_a – amplitude of normal stress induced by bending, τ_a – amplitude of shear stress induced by torsion, ω – angular frequency, φ – angle of phase shift, t - time.
In the discussed model, the course of normal stress $\sigma_{xx}(t)$ refers to stress induced by bending, while $\tau_{xy}(t)$ refers to torsion-induced stress. The next step involves determination of the orientation angle of the critical plane, which can be done using one of three established methods: weight functions, damage accumulation or variance. In this paper, the orientation of the critical plane was determined using damage accumulation method. If the criterion proposed by Carpinteri et al. [2] is used, the inclination angle of the critical plane is increased by the angle

$$\beta = \frac{3}{2}\left[1 - \left(\frac{1}{B_2}\right)^2\right] 45° \tag{3}$$

with respect to the angle determined by maximum normal stress, where:

$$B_2 = \frac{\sigma_{af}}{\tau_{af}} \tag{4}$$

where σ_{af}, τ_{af} are fatigue limits for bending and torsion respectively.
The relationship (4) was proposed for some selected constructional materials, and the group for which this relationship is constant was determined. In such a case, hypotheses allow to calculate the fatigue life. As for other materials, there is no one universal criterion of fatigue life calculation because it is necessary to include variation of the ratio σ_a/τ_a depending on a number of cycles to the fatigue failure [5, 16].
There is a number of multiaxial fatigue criteria. Here, we are discussing the group based on the critical plane concept. Macha [8] has formulated the criterion of maximum normal and shear stress in fracture plane which can be generalised for the scope of random loading of numerous criteria. The general form can be written down as

$$\sigma_{eq}(t) = B\tau_{\eta s}(t) + K\sigma_{\eta}(t) \tag{5}$$

where: B, K – constants used for selection of specific criterion form [7]
In this paper, in order to verify the highest conformity of results, three different criteria of multiaxial fatigue were used:
1. Criterion in the maximum normal stress plane, in the following form

$$\sigma_{eq}(t) = B_1\tau_{\eta s}(t) + \sigma_{\eta}(t) \tag{6}$$

where: B_1 – constant depending on material type, $\sigma_\eta(t)$ is the course of normal stress orientated at angle α towards σ_{xx}, expressed by the following equation

$$\sigma_{\eta}(t) = \sigma_{xx}(t)\cos^2\alpha + \tau_{xy}(t)\sin 2\alpha \tag{7}$$

Whereas $\tau_{\eta s}(t)$ is the course of shear stress

$$\tau_{\eta s}(t) = -\frac{1}{2}\sigma_{xx}(t)\sin 2\alpha + \tau_{xy}(t)\cos 2\alpha \tag{8}$$

where:

$$\alpha = \alpha_\eta + \beta \tag{9}$$

α_η is the angle defined by normal stress using the damage accumulation method or by finding an angle, for which normal stress variance reaches maximum [13, 15]

$$\mu_{\eta\eta}(\alpha_n) = \frac{1}{T_o}\int_o^{T_o} \sigma_\eta^2(t)\,dt \tag{10}$$

where T_o is time of observation, in constant amplitude loading it is one cycle.

2. Criterion in the maximum shear stress plane, in the following form

$$\sigma_{eq}(t) = B_2 \tau_{\eta s}(t) + (2 - B_2)\sigma_\eta(t) \tag{11}$$

where in general case:

- $B_2 = \sigma_a(N_{fi}) / \tau_a(N_{fi})$ (12)

- N_{fi} is number of cycles, for which amplitude ratio is defined. When characteristics are parallel, we take fatigue limit as defined in formula (4) [6, 16].

3. Criterion using determination of critical plane orientation according to the Carpinteri et al. method as defined in (3), where weighing factors can be defined as:

$$B = \frac{B_2 - \dfrac{\sin(90^o + 2\beta)}{\cos^2\beta}}{\dfrac{\sin 2\beta \sin(90^o + 2\beta)}{2\cos^2\beta} + \cos(90^o + 2\beta)} \tag{13}$$

$$K = \frac{2 + B\sin 2\beta}{2\cos^2\beta} \tag{14}$$

The final step is the calculation of fatigue strength. For fixed amplitude loadings (cyclical), the fatigue strength is calculated using Basquin's fatigue characteristics, in compliance with the relevant ASTM standard [1]. The formula for calculation strength under cyclical loading is expressed as

$$N_{cal} = 10^{A - m \lg \sigma_{eg,a}} \tag{15}$$

ANALYSED MATERIALS

The analysis used the results of fatigue tests of the following materials: two aluminium alloys: PA4 (6082) [10], PA6 (2017A) [3] , 10HNAP [11] and 30CrNiMo8 [14] steels, GGG40 cast iron [9], and brass CuZn40Pb2 [5]. The results were also used to calculate the regression equations for oscillatory bending (or unixial push-pull), as per the ASTM recommendations [1], in the following form

$$\log N_f = A_\sigma + m_\sigma \log\sigma_a. \tag{16}$$

For bilateral torsion, the regression equation takes the form of

$$\log N_f = A_\tau + m_\tau \log\tau_a, \tag{17}$$

where:

A_σ, m_σ, A_τ, m_τ - coefficients of regression equation for oscillatory bending and bilateral torsion, respectively.

Tab. 1 lists the values of coefficients of regression equation for the analysed materials.

Fig. 1 shows fatigue diagram for oscillatory bending and bilateral torsion on the example of the PA4 aluminum alloy.

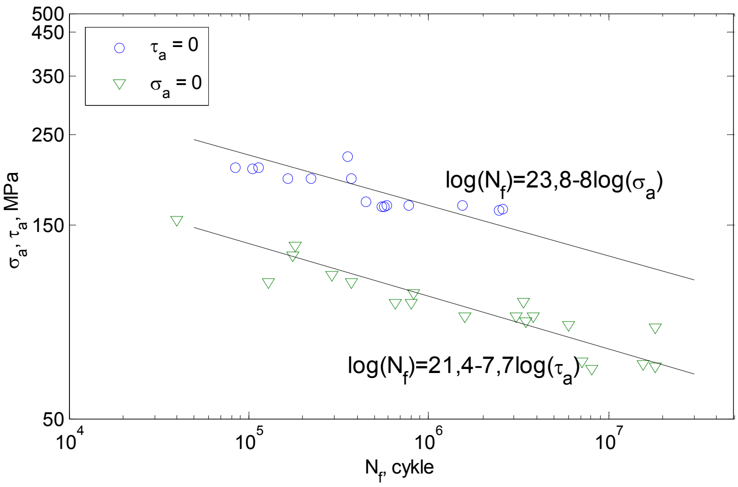
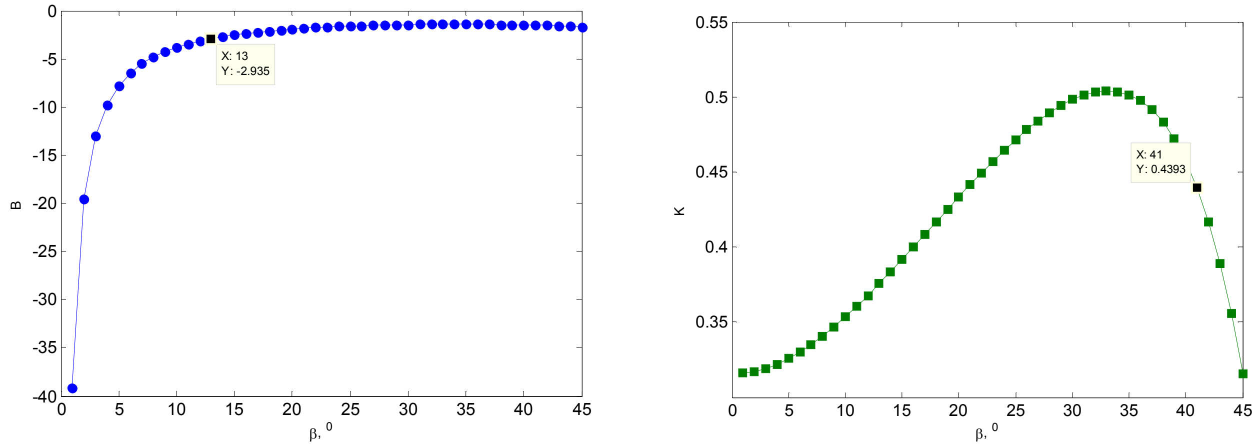


Figure 1: Fatigue diagram for oscillatory bending and bilateral torsion for the 6082-T6 aluminum alloy (where τ_a, σ_a are stress amplitudes generated by torsional moment and bending moment, respectively).

Material	Bending		Torsion		N_{fi}	σ_a/τ_a (N_{fi})
	A_σ	m_σ	A_τ	m_τ	cycles	
PA6 (2017A)	21.87	-7.03	19.94	-6.87	2000000	1.696
GGG40	32.39	-10.95	35.48	-12.41	1000000	1.11
10HNAP	30.88*	- 9.5*	25.28	- 8.2	2000000	1.874
PA4 (6082)	23.8	-8.0	21.4	- 7.7	2000000	1.68
30CrNiMo8	27.54	8.05	69.56	24.62	100000	1.5
CuZn40Pb2	19.99	5.86	45.3	17.17	1000000	0.92

Table 1: Coefficients of regression equation for analysed materials.

$\in < 0°, 45° >$. For each of the 46 angles calculated parameters B and K in accordance with the formulas (13) and (14). Fig. 2 presents B and K constants depending on the angle β for the PA4 aluminum alloy. $N_{cal} = 10^{A - m \lg \sigma_{eq,a}}$.



(a) (b)

Figure 2: The dependence of the parameter a) B, b) K from the angle β for aluminum alloy 6082 (PA4).

VERIFICATIONS OF PROPOSED CRITERIA AND ANALYSIS OF OBTAINED RESULTS

In order to perform the correct analysis of the fatigue life scatter, the logarithmic dependence of the ratios of experimental and calculation strength should be used. A new method of determination the fatigue life scatter has been proposed by [12], defined as the root mean square error:

$$E = \sqrt{\frac{\sum_{i=1}^{n} \log^2 \frac{N_{\exp}}{N_{cal}}}{n}} \tag{18}$$

Therefore, the scatter can be determined as:

$$T = 10^E \tag{19}$$

Tab. 2 compares plane rotation angles according to the Carpinteri et al. method. Moreover, the angle β was found for minimum scatter according to the formula (19). Fig. 3 shows the relationship between scatter value (19) and the angle β for the selected PA4 aluminum angle. For this alloy, the least scatter (global minimum) was achieved for β=42[0] (T=2.366). Additionally, local minimum was obtained for β =19[0] (T=2.391).

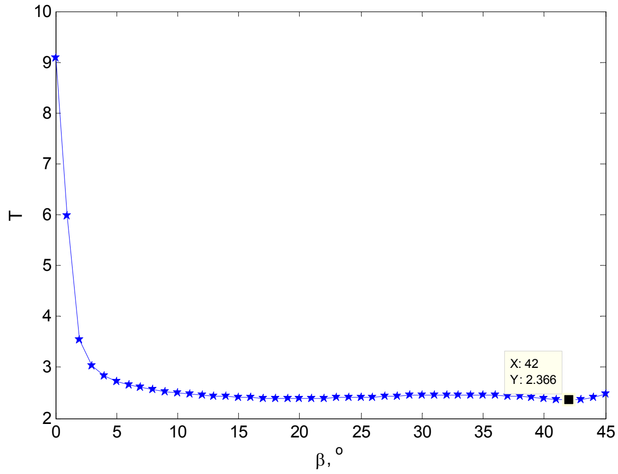


Figure 3: Relationship between scatter values T (19) and the angle β.

Material	β_C (3)	T_{min}	β (T_{min})	β (21)
PA6	44	1.9972	44	42
GGG40	13	2.583	1	16
10HNAP	*	1.932	43	44
PA4	44	2.366	42	42
30CrNiMo8	38	1.992	40	39
CuZn40Pb2	**	4.335	11	→0

Table 2: The values of rotation angles β and minimum scatters of fatigue life (*out of range > $\sqrt{3}$; ** out of range <1

The Carpinteri et al. method concerning determination of the angle β is based on the assumption that we deal with materials ranging from resilient-brittle to resilient-plastic state. Considering this, the Author assumes that in the first case coefficient B_2 is 1, and in the second extreme case its value is $\sqrt{3}$. Therefore, it has been assumed that the plasticization criterion according to the Heber-Mises-Hencke hypothesis is right in the boundary case for materials in resilient-plastic state. As we see, this ratio (Tab. 1) for the 10HNAP steel exceeds that value. Considering this, it has been proposed to introduce the boundary plasticization value according to the Tresca hypothesis (max. {B} = 2). Thus, the formula analogical to (3) is as follows:

$$\beta = \frac{4}{3}\left[1-\left(\frac{1}{B_2}\right)^2\right]45° \tag{20}$$

When we analyse the data shown in Fig. 4, we can see that the points defined as scatter minimum for individual materials, lie along the more convex curve. Considering this, a new curve has been proposed, satisfying the same boundary conditions as the formula (20) - with the arguments fitting within range <0, 2>, in the following form:

$$\beta = \frac{16}{15}\left[1-\left(\frac{1}{B_2}\right)^4\right]45° \tag{21}$$

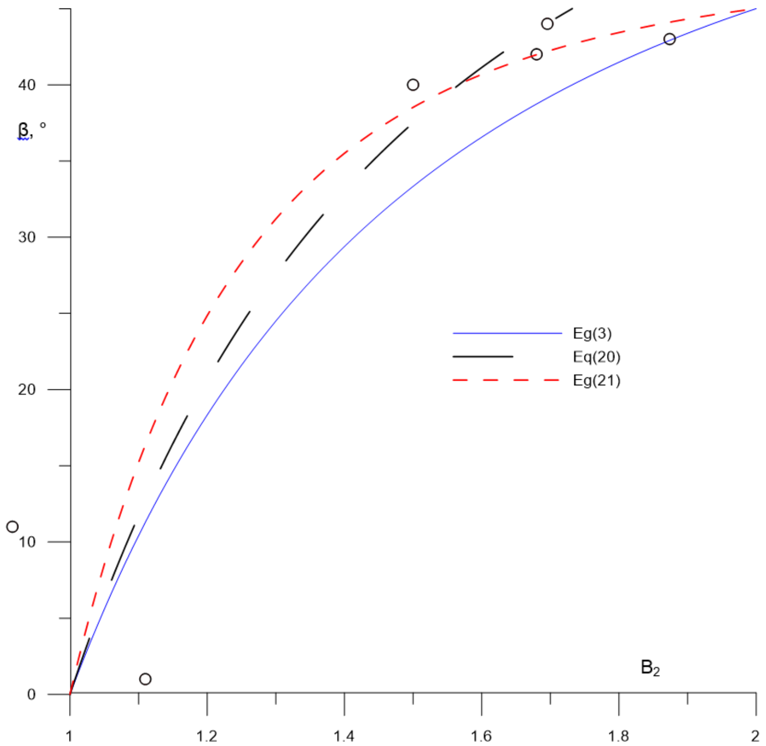


Figure 4: The relationship between the angle β and the parameter B_2 according to different models.

Conclusions

Analysing the results of fatigue tests and calculations, it can be concluded that:

1. The multiaxial fatigue criterion proposed for life estimation is applicable to wide range of materials from resilient-brittle to resilient-plastic state.
2. The Carpinteri et al. hypothesis has proven to be right. The hypothesis concerns orientation rotation for the critical plane defined by normal stresses by the angle dependent on the ratio of fatigue boundaries for bending and torsion.
3. The Tresci criterion has been taken as the boundary condition for materials in resilient-plastic state. The new form of formula for the critical plane orientation rotation angle has been proposed on the basis of this condition.
4. Further verification is required for other materials and other loading conditions.

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